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r- Lorentzian Para Sasakaian Manifolds With Semi Symmetric Recurrent Metric Connection

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Abstract

Several author as Agashe and Chafle [1], Sengupta, De. U.C, Binh [4] and other introduced semi symmetric non metri connection in different way. In this paper we have studied r-LP sasakian manifold with special semi-symmetric recurrent metric connection [2] and discuss it existence in LP sasakian manifold. In section 3 we establish the relation between the Riemannian connection and special semi-symmetric recurrent metric connection on r-LP sasakian manifold [4]. The section 4 deals with ξ_p-conformly flat and φconcircularly flat of n dimensional r-LP sasakian manifold and we proved that ξ_p-conformly flatness with special semi-symmetric recurrent metric connection and Riemannian manifold coincide.

Keywords: ξ_p-conformly flat, φ concircularly , Riemannian manifold, r-LP sasakian manifold, special semi-symmetric recurrent metric connection.

1. Introduction

An n-dimensional differentiable manifold M^n is a r-lorentzian para- sasakian manifold if it admits tensor field of type (1, 1), a contravariant vector fieldξ_p, a covariant vector field $\eta^p$and a lorentzian metric g satisfying;

$$\varphi^2 X = X + \eta^p(X)\xi_p \tag{1.1}$$

$$\eta^p(\xi_p) = -1 \tag{1.2}$$

$$g(\varphi X, \varphi Y) = g(X,Y) + \eta^p(X)\eta^p(Y) \tag{1.3}$$

$$g(X, \xi_p) = \eta^p(X) \tag{1.4}$$

$$(D_X\varphi)(Y) = g(X,Y)\xi_p + \eta^p(Y)X + 2\eta^p(X)\eta^p(Y)\xi_p \tag{1.5}$$

$$D_X\xi_p = \varphi X \tag{1.6}$$

For arbitrary vector field and Y , where D denotes the operator of covariant differentiation with respect to r- lorentzian metric g [5].

In a r-LP sasakian manifold with structure $(\varphi, \xi_p, \eta^p, g)$ the following relation hold.

$$(a)\varphi(\xi_p) = 0 (b)\eta^p(\varphi X) = 0 (c)\text{rank}\varphi = n - 1 \quad (1.7)$$

Let us put $F(X, Y) = g(\varphi X, Y)$ (1.8)

The tensor field F is symmetric (0,2) tensor field i.e. $F(X, Y) = F(Y, X)$ (1.9)

And $(D_X \eta^p)(Y) = F(X, Y) = g(\varphi X, Y)$ (1.10)

Also in a r-LP sasakian manifold the following relation holds;

$$g(R(X, Y)Z, \xi_p) = \eta^p(R(X, Y)Z) = g(Y, Z)\eta^p(X) - g(X, Z)\eta^p(Y) \quad (1.11)$$

And $S(Y, \xi_p) = (n - 1)\eta^p(X)$ (1.12)

For any vector field X, Y, Z where $R(X, Y)Z$is the Riemannian curvature tensor and S is the Ricci tensor.

Let (M^n, g) be an r-LP sasakian manifold with Levi-Civita connection D. We define a linear connection $\overline{D}$ on M^n by

$$\overline{D}_X Y = D_X Y - \eta^p(X)Y \quad (1.13)$$

Where η^p is 1-form associated with vector field ξ_p on M^n, given by

$$g(X, \xi_p) = \eta^p(X) \quad (1.14)$$

Using (1.13) the torsion tensor $\overline{T}$ on M^n with respect to connection $\overline{D}$ is given by

$$\overline{T}(X, Y) = \overline{D}_X Y - \overline{D}_Y X - [X, Y] = \eta^p(Y)X - \eta^p(X)Y \quad (1.15)$$

A linear connection satisfying (1.15) is called semi symmetric connection.

Further from (1.13), we have

$$(\overline{D}_X g)(Y, Z) = 2\eta^p(X)g(Y, Z) \quad (1.16)$$

A linear connection satisfying (1.16) is called semi symmetric recurrent metric connection. The word special is used to distinguish it from other connection.

2. Existence of Special Semi- Symmetric Recurrent Metric Connection

Let $\overline{D}$ be a linear connection in M^n, given by $\overline{D}_X Y = D_X Y + H(X, Y)$ (2.1)

Where H is a tensor of type (1, 2).

Now, we determine the tensor field H such that $\overline{D}$satisfies (1.15) and (1.16).

From (2.1), we have

$$\overline{T}(X, Y) = H(X, Y) - H(Y, X) \tag{2.2}$$

Let $G(X, Y, Z) = (\overline{D}_X g)(Y, Z)$ (2.3)

Then $g(H(X, Y), Z) + g(H(X, Z), Y) = G(X, Y, Z)$ (2.4)

From (2.1), (2.2) and (2.4), we have

$$\begin{gathered} g(\overline{T}(X, Y), Z) + g(\overline{T}(Z, X), Y) + g(\overline{T}(Z, Y), X) \\ g(H(X, Y), Z) - g(H(Y, X), Z) + g(H(Z, X), Y) - g(H(X, Z), Y) \\ + g(H(Z, Y), X) - g(H(Y, Z), X) \\ 2g(H(X, Y), Z) + 2\eta^p(X) g(Y, Z) - 2\eta^p(Z) g(X, Y) + 2\eta^p(Y) g(X, Z) \end{gathered}$$

Or

$$H(X, Y) = \frac{1}{2}\{\overline{T}(X, Y) + \overline{T'}(X, Y) + \overline{T'}(Y, X)\} - \eta^p(X) Y - \eta^p(Y) X + g(X, Y)\xi_p \tag{2.5}$$

Where $g(\overline{T'}(X, Y), Z) = g(\overline{T}(Z, X), Y)$ (2.6)

Using (1.5), (2.6) we get

$$\overline{T'}(X, Y) = \eta^p(X) Y - g(X, Y)\xi_p \tag{2.7}$$

Then in view of (1.15), (2.5) and (2.7), we get $H(X, Y) = -\eta^p(X) Y$

This implies $\overline{D}_X Y = D_X Y - \eta^p(X) Y$

Conversely, a connection $\overline{D}$given by (1.13), satisfies (1.15) and (1.16) show that $\overline{D}$ is special semi symmetric recurrent metric connection.

So we state the following theorem.

Theorem 2.1 Let (M^n, g) be an r- LP sasakian manifold with lorentzian Para contact metric structure $(\varphi, \xi_p, \eta^p, g)$ admits a special semi – symmetric connection which is given by $\overline{D}_X Y = D_X Y - \eta^p(X) Y$

3. Curvature tensor of M^n With Respect To Special Semi- Symmetric Recurrent Metric Connection $\overline{D}$

The Curvature tensor of M^n with respect to special semi-symmetric recurrent metric connection $\overline{D}$is given by $\overline{R}(X,Y,Z) = \overline{D}_X\overline{D}_Y Z - \overline{D}_Y\overline{D}_X Z - \overline{D}_{[X,Y]}Z$

Using (1.13) and (1.10) in above we have

$$\overline{R}(X,Y,Z) = R(X,Y,Z) \tag{3.1}$$

Hence we conclude.

Proposition 3.1 The Curvature tensor of M^n with respect to special semi-symmetric recurrent metric metric connection $\overline{D}$coincide with the curvature tensor of taking the inner product of (3.1) with W, we have

$$\overline{R}(X,Y,Z,W) = R(X,Y,Z,W) \tag{3.2}$$

Where $\overline{R}(X,Y,Z) = g\big(\overline{R}(X,Y,Z),W\big)$

From (3.2), we have $\overline{R}(X,Y,Z,W) = -R(Y,X,Z,W)$ (3.3)

$$\overline{R}(X,Y,Z,W) = -R(X,Y,W,Z) \tag{3.4}$$

Combining above two relation, we have $\overline{R}(X,Y,Z,W) = -R(Y,X,W,Z)$ (3.5)

We also have, $\overline{R}(X,Y,Z) + \overline{R}(Y,Z,X) + \overline{R}(Z,X,Y) = 0$ (3.6)

This is the Binachi first identity for$\overline{D}$.

Hence we conclude that the curvature tensor of M^n with respect to special semi- symmetric recurrent metric metric connection $\overline{D}$ satisfies the first Binachi identity. Contracting (3.2) over X and W, we obtain

$$\overline{S}(Y,Z) = S(Y,\text{YZ}) \tag{3.7}$$

Where $\overline{S}$ and S denote the Ricci tensor of the connection $\overline{D}$ and D respectively.

From (3.7) we obtain a relation between the scalar curvature of M^n with respect to the Riemannian connection and special semi- symmetric recurrent metric connection which is given by $\overline{r} = r$ (3.8)

So we have following:

Proposition 3.2 Form dimensional r- LP sasakian manifold with special semi-symmetric recurrent metric connection $\overline{D}$

(1) The curvature tensor $\overline{R}$ is given by (3.1)
(2) Ricci tensor $\overline{S}$is given by (3.7)
(3) $\overline{r} = r$

4. CONCIRCULAR CURVATURE TENSOR OF r- LP SASAKIAN MANIFOLD WITH RESPECT TO SPECIAL SEMI- SYMMETRIC RECURRENT METRIC CONNECTION

Analogous to the definition of concircular curvature tensor in a Riemannian manifold we define concircular curvature tensor with respect to the special semi symmetric recurrent metric connection $\overline{D}$ as

$$\overline{C}(X,Y,Z) = \overline{R}(X,Y,Z) - \frac{\tau}{n(n-1)}\{g(Y,Z)X - g(X,Z)Y\} \qquad (4.1)$$

Using (3.1) and (3.7) in (4.1), we have $\overline{C}(X,Y,Z) = C(X,Y,Z)$ (4.2)

So we have

Proposition 4.1 $\overline{C}(X,Y,Z) = C(X,Y,Z)$that is manifold coincide with Riemannian Manifold.

The notion of an ξ_p-conformaly flat r-contact manifold was given by Zhen, Cabrezizo and Fermander [3]. In an analogous we define an ξ_p-conformaly flat n- dimensional r- LP sasakian manifold.

Definition 4.2 An n-dimensional r- LP sasakian manifold is called ξ_p-conformaly flat if the condition $\overline{C}(X,Y)\xi_p = 0$ holds onM^n.

From (4.2) it is clear that$\overline{C}(X,Y)\xi_p = C(X,Y)\xi_p$.

So we have the following theorem.

Theorem 4.1 In an n-dimensional r-LP Sasakian manifold, an ξ_p-conformaly flatness with respect to special semi- symmetric recurrent metric connection and Riemannian connection coincide.

Definition 4.3 An n- dimensional r-LP Sasakian manifold satisfying the condition

$$\varphi^2\overline{C}(\varphi X, \varphi Y)\varphi Z = 0 \qquad (4.3)$$

is called φ-concircularly flat.

Let us suppose that M^n be n-dimensional φ-concircularly flat r-LP sasakian manifold with respect to special semi- symmetric recurrent metric connection. It can easily be seen that $\varphi^2\overline{C}(\varphi X, \varphi Y)\varphi Z = 0$ if and only if

$$g\left(\overline{C}(\varphi X, \varphi Y)\varphi Z, \varphi W\right) = 0 \quad (4.4)$$

For all X, Y, Z, W on T(M).

Using (4.1), φ-concircularly flat means

$$g\left(\overline{R}(\varphi X, \varphi Y)\varphi Z, \varphi W\right) = \frac{\tau}{n(n-1)}\{g(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)g(\varphi Y, \varphi W)\} \quad (4.5)$$

Let $\{e_1, e_2 \ldots\ldots, \xi_p\}$ be a local orthogonal basis of the vector in M^n using the fact that $\{\varphi e_1, \varphi e_2 \ldots\ldots, \xi_p\}$ is also a local orthogonal basis, putting $X = W = e_i$ in (4.5) and summing with respect to I, we have

$$S(\varphi Y, \varphi Z) = \frac{\tau}{n(n-1)}\{g(\varphi Y, \varphi Z)\} \quad (4.6)$$

Putting $Y = \varphi Y, Z = \varphi Z$ in (4.6) and using the fact S is symmetric, we have $g\left(\overline{R}(\varphi X, \varphi Y)\varphi Z, \varphi W\right) = 0$

Hence we have

Theorem 4.2 An n- dimensional r-LP sasakian manifold is φ-concircularly flat with respect to special semi- symmetric recurrent metric connection manifold coincide with Riemannian Manifold.

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